Chapter-IV

Planetary Boundary Layer (PBL)

A Brief essay on PBL:

PBL is the lower most portion of the atmosphere, adjacent to the earth's surface, where maximum interaction between the Earth surface and the atmosphere takes place and thereby maximum exchange of Physical properties like momentum, heat, moisture etc., are taking place.

Exchange of physical properties in the PBL is done by turbulent motion, which is a characteristic feature of PBL. Turbulent motion may be convectively generated or it may be mechanically generated.

If the lapse rate near the surface is super adiabatic, then PBL becomes positively Buoyant, which is favourable for convective motion. In such case PBL is characterized by convective turbulence. Generally over tropical oceanic region with high sea surface temperature this convective turbulence occurs.

If the lapse rate near the surface is sub adiabatic then the PBL is negatively buoyant and it is not favourable for convective turbulence. But in such case, if there is vertical shear of horizontal wind, then Vortex (cyclonic or anti cyclonic) sets in, in the vertical planes in PBL, as shown in the adjacent fig 2b. This vortex motion causes turbulence in the PBL, known as mechanical turbulence.

If the PBL is positively buoyant as well as, if vertical shear of the horizontal wind exists, then both convective and Mechanical turbulence exits in the PBL. The depth of the PBL is determined by the maximum vertical extent to which the turbulent motion exists in PBL. On average it varies from few cms to few kms. In case of thunderstorms PBL may extend up to tropopause.

Generally at a place on a day depth of PBL is maximum at noon and in a season it is maximum during summer.

Division of the PBL into different sub layers:

The PBL may be sub divided into three different sections, viz viscous sub layer, the surface layer and the Ekmann layer or entrainment layer or the transition layer.

Viscous layer is defined as the layer near the ground, where the transfer of physical quantities by molecular motions becomes important. In this layer frictional force is comparable with PGF.

The surface layer extends from z = z_0 (roughness length) to $z = z_s$ with z_s, the top of the surface layer, usually varying from 10 m to 100 m. In this layer sub grid scale fluxes of momentum (eddy stress) and frictional forces are comparable with PGF.

The last layer is the Ekmann layer is defined to occur from z_s to z_i, which ranges from 100 m or so to several kilometers or more. Above the surface layer, the mean wind changes direction with height and approaches to free stream velocity at the height z as the sub grid scale fluxes decrease in magnitude. In this layer both the COF and Eddy stress are comparable with PGF.

Boussinesq approximation: According to this approximation density may be treated as constant everywhere in the governing equations except in the vertical momentum equation, where it is coupled with Buoyancy term. Physically this approximation says that the variation of density in the horizontal direction is insignificant as compared to that in the vertical direction.

Governing equations in the PBL: Governing equations in the PBL, following adiabatic and Boussinesq approximation, are given below:

$$\frac{\partial u}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)u = \frac{-1}{\rho_0}\frac{\partial p}{\partial x} + fv + F_x \text{ ....(4.1)}$$

$$\frac{\partial v}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)v = \frac{-1}{\rho_0}\frac{\partial p}{\partial y} - fu + F_y \text{ ......(4.2)}$$

$$\frac{\partial w}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)w = \frac{-1}{\rho_0}\frac{\partial p}{\partial z} - g\frac{\theta}{\theta_0} + F_z \text{ ......(4.3)}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ ....(4.4)}$$

$$\frac{\partial \theta}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\theta = 0 \text{. ......(4.5)}$$

Concepts of mean motion and eddy motion in the PBL & Reynolds averaging technique.

In the PBL both the mean motion and the eddy motion are very important. Hence it is required to have equations for both motion.

To distinguish these two, Reynold devised an averaging method, which is discussed below:

Let us consider any field 'S'at a synoptic hour T. Let S_{Obs} bet he observed value of 'S' at time T hrs. Now to find out the contribution from mean and eddy motion towards 'S', we have to take a number of observations of 'S' during the time interval $\left(T - \frac{\tau}{2}, T + \frac{\tau}{2}\right)$. Suppose during the above period we have 'n' observations. Viz., $S_1, S_2, \ldots.., S_n$ of S. Then $\overline{S} = \int_{T-\frac{\tau}{2}}^{T-\frac{\tau}{2}} s\, dt \approx \frac{1}{n}\sum_1^n S_i$ is called the mean part of 'S' at T, and $S' = \left(S_{obs} - \overline{S}\right)$ is called the eddy part of S at time T hrs. This Eddy part is due to turbulent eddy motion in the PBL. The quantity 'τ' is called averaging interval. While choosing 'τ' the following precautions are necessary to take:

a) It should not be too small to miss the trend in mean motion.

b) It should not be too large that eddies filtered out.

For two arbitrary quantity, say, α and β, we have, $\alpha = \overline{\alpha} + \alpha'$ and $\beta = \overline{\beta} + \beta'$. Hence, $\overline{\alpha\beta} = \overline{\alpha}\overline{\beta} + \overline{\alpha'\beta'}$. The last term is known as eddy co-variance.

Concept of Eddy flux and Eddy flux divergence/ convergence:

Flux of any field refers to the transport of that field in unit time across unit area. Hence flux of a field, say S, is $S\vec{V}$, $\vec{V}$ being wind velocity.

Eddy flux, thus refers to the transport of some field by eddy wind. If u', v', w' are the components of eddy wind, then eddy wind vector is given by $\vec{V}' = (\hat{i}u' + \hat{j}v' + \hat{k}w')$, then eddy flux of a quantity S is $S\vec{V}'$.

Flux divergence/convergence physically refers to the dispersion or accumulation of the field after being transported. Mathematically it is expressed as $\vec{\nabla}.(S\vec{V}')$.

In the mean equations of motion some new terms have appeared.

These terms are known as eddy flux convergence of eddy momentum. Physically they may interpreted as follows:

Let us consider, the eddy zonal momentum (u') is being transported by all the three components u', v', w' of eddy wind. Now eddy zonal momentum transported by these components in unit time across unit area are respectively $u'u', u'v'$ *and* $u'w'$.

The first one is along $\hat{i}$ direction, second one in $\hat{j}$ direction and third one in $\hat{k}$ direction. Thus at any point transport of u' may be expressed as the vector $\left(u'\vec{V}'\right)$.

After being transported, the eddy u momentum is being accumulated, which is expressed as $-\vec{\nabla}.\left(u'\vec{V}'\right)$. This term is called eddy flux convergence of u'. Thus, this much eddy zonal momentum is being added to the existing mean zonal momentum u, causing a change in u. Thus this term has appeared in the zonal momentum equation for the mean flow. Similarly one can argue for the existence of the other eddy flux convergence terms.

Governing equations for mean motion: To obtain the equations for mean flow, we first need to express terms like, $(\vec{V}.\vec{\nabla})u$ in flux form.

We know that, $(\vec{V}.\vec{\nabla})u = \vec{\nabla}.(u\vec{V}) - u\vec{\nabla}.\vec{V}$. Again following Boussinesq approximation, $\vec{\nabla}.\vec{V} = 0$.

Hence, $(\vec{V}.\vec{\nabla})u = \vec{\nabla}.(u\vec{V}) = \vec{\nabla}.\left[(\overline{u} + u')(\overline{\vec{V}} + \vec{V}')\right] = \vec{\nabla}.(\overline{u}\overline{\vec{V}}) + \vec{\nabla}.(\overline{u}\vec{V}') + \vec{\nabla}.(u'\overline{\vec{V}}) + \vec{\nabla}.(u'\vec{V}')$

Taking Reynolds average, we have,

$\overline{(\vec{V}.\vec{\nabla})u} = \vec{\nabla}.(\overline{u}\overline{\vec{V}}) + \vec{\nabla}.(\overline{u'\vec{V}'})$

Again $\overline{\frac{\partial u}{\partial t}} = \overline{\frac{\partial \overline{u}}{\partial t} + \frac{\partial u'}{\partial t}} = \frac{\partial \overline{u}}{\partial t}$ and, $\frac{\partial \overline{u}}{\partial t} + \vec{\nabla}.(\overline{u}\overline{\vec{V}}) = \frac{\partial \overline{u}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{u} + \overline{u}(\vec{\nabla}.\overline{\vec{V}}) = \frac{\partial \overline{u}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{u}$

Hence, the governing equations for mean flow are

$$\frac{\partial \overline{u}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{u} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + f\overline{v} + \overline{F}_x - \vec{\nabla}.(\overline{u'\vec{V}'}) \;\ldots\ldots\; (4.6)$$

$$\frac{\partial \overline{v}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{v} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial y} - f\overline{u} + \overline{F}_y - \vec{\nabla}.(\overline{v'\vec{V}'}) \ldots\ldots\ldots (4.7)$$

$$\frac{\partial \overline{w}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{w} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial z} - g\frac{\overline{\theta}}{\theta_0} + \overline{F}_z - \vec{\nabla}.(\overline{w'\vec{V}'}) \ldots\ldots (4.8)$$

$$\frac{\partial \overline{\theta}}{\partial t} + (\overline{\vec{V}}.\vec{\nabla})\overline{\theta} = -\vec{\nabla}.(\overline{\theta'\vec{V}'}) \ldots\ldots (4.9)$$

$$\vec{\nabla}.\overline{\vec{V}} = 0 \ldots\ldots (4.10)$$

Turbulent Kinetic Energy Equation

Turbulent Kinetic Energy equation is obtained from the equations of motion , in component form, for turbulent motion, which can be obtained by subtracting equations 4.6, 4.7 & 4.8 from 4.1, 4.2 & 4.3 respectively. Then the subtracted equations are multiplied by u', v', w' respectively, then they are added and then taking Reynolds average we obtain required TKE equation

$$\frac{\partial(\overline{TKE})}{\partial t} = MP + BPL + TR - \varepsilon \ldots\ldots (4.11)$$

Where, $BPL = \frac{g}{\theta_0}\overline{w'\,\theta'} = -N^2\ \overline{\xi'\,w'}$, and $MP = -\left(\overline{w'\vec{V}'}\right) \bullet \frac{\partial \vec{\overline{V}}}{\partial z}$.

Here, BPL stands for Buoyancy production or loss, MP stands for Mechanical production, N^2 stands for square of Brunt-Vaisalla frequency and ξ' is eddy vertical displacement. TR stands for redistribution by transport and pressure forces and ε represents frictional dissipation which is always positive reflecting the dissipation of the smallest scale of turbulence by molecular viscosity.

In our introduction we have mentioned that generally turbulence in PBL is either convectively or mechanically generated.

Now we shall see that, for convectively generated turbulence, through BPL term eddies are being supplied K.E.

Effect of Buoyancy production or loss (BPL) term : To examine, the effect of this term, we shall consider three conditions viz., When atmosphere is stably stratified, When atmosphere is unstably stratified and When atmosphere is neutrally stratified.

First of all we must note that eddy co-variance between eddy vertical velocity and vertical displacement must be positive, as the former one must be upward or downward if the later is so. If the Atmosphere is stably stratified, then we know that N^2 is positive. Hence in that case BPL must be negative. Thus convective turbulence is suppressed in a stably stratified PBL. Similarly one can show that in an unstably stratified PBL ($N^2 < 0$), BPL is positive and convective turbulence is sustained. Finally if the PBL is neutrally stratified, then $N^2 = 0$. So BPL = 0, hence Convective turbulence is neither generated nor sustained.

Effect of Mechanical Production (MP) term:

In the introduction it was shown qualitatively that Mechanically generated turbulence can occur only if there is a vertical shear (either cyclonic or anticyclonic) of the horizontal wind.

Now we can discuss the MP term and see how it is significant for mechanically generated turbulence. First term of MP represents the vertical eddy flux of eddy horizontal momentum and the second one is vertical shear of eddy horizontal momentum (i.e., vertical gradient of the components of mean horizontal wind). Qualitatively one can argue that if the vertical gradient of any quantity is positive (i.e., upward), then eddy transport of that quantity has to be downward and the vice-versa. Thus we see that vertical gradient of the mean and vertical eddy transports are opposite to each other. As a result of which MP is always positive, provided there is a vertical shear of mean horizontal wind. Hence in any case, due to MP term TKE increases with time and Turbulence is sustained. Thus, whenever there is vertical shear of the mean horizontal wind, then mechanically generated turbulence occurred.

Now, we consider a typical situation, when PBL is stably stratified is and there exists vertical shear of mean horizontal wind. In such situation BPL term inhibits turbulence and MP term enhances turbulence. In this situation it is difficult to say whether their combined effect is to suppress turbulence or to sustain turbulence. It has been found empirically that to maintain the turbulence, MP should exceed four times the BPL. This condition is measured by a quantity called the flux Richardson number (Rf), which is defined by $R\,f = -\frac{BPL}{MP}$. If the PBL is unstably stratified then $R\,f < 0$ and turbulence is sustained by convection, as mentioned earlier. If PBL is stably stratified, then $R\,f > 0$. In such case $R\,f$ must be less than 0.25 to sustain the turbulence. Thus $R\,f$ should be less than ¼ to maintain turbulence in a stably stratified PBL by wind shear.

Sub-grid scale Physical processes: A Physical process whose spatial dimension is less than the grid scale, is known as sub-grid scale Physical process. Sub grid scale physical processes may be taken place in a smaller area, but its impact may be significant on the large scale flow. Tto illustrate it we give the following example:-

Let a small region be conditionally unstable. Now, in this region moist convection is taking place i.e. moist air parcel is rising. Now at a level where all the moistures inside the air parcel has condensed releasing latent heat. That released Latent heat, which may be due to a sub grid scale physical process, viz, moist convection, will in turn heat up the atmosphere at that level, which may affect the temperature at the grid point.

Thus, we see that though a physical process is capable of escaping from being caught at the grid points, it affects the field value at the grids. So if the effects of sub grid scale physical process cannot be incorporated in the NWP model, then forecast issued by NWP is definitely going to be wrong.

Parameterization of sub grid scale physical processes in the PBL.

To parameterize the sub grid scale physical processes in the PBL first we should know that what sub grid scale physical process is taking place in the PBL. The only sub grid scale physical process taking place in PBL is the vertical eddy transport (Order of magnitude of the horizontal eddy transport is very less compared to the vertical eddy transport.).

Again the method of parameterization of eddy transport depends on the nature of PBL.

The vertical profile of the horizontal components of the mean wind using the paramaterisation scheme.

For the special case of horizontally homogeneous turbulence above the viscous sub-layer, molecular viscosity and horizontal turbulent flux divergence terms can be neglected. The mean horizontal equations of motion then become

$$\frac{\partial \overline{u}}{\partial t} + \left(\vec{\overline{V}}.\vec{\nabla}\right)\overline{u} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \frac{\partial(\overline{u'w'})}{\partial z} \;\ldots\ldots(4.12)$$

$$\frac{\partial \overline{v}}{\partial t} + \left(\vec{\overline{V}}.\vec{\nabla}\right)\overline{v} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial y} - f\overline{u} - \frac{\partial(\overline{v'w'})}{\partial z} \;\ldots\ldots(4.13)$$

For the mid latitude synoptic scale system, we know that to a first approximation, acceleration term may be neglected as compared to the Coriolis force and Pressure gradient force terms. Out side the PBL, this approximation simply results into geostrophic balance. Inside PBL also the acceleration terms are still small compared to the Coriolis force and Pressure gradient force terms, but the turbulent flux term must be retained. So, there is an approximate balance in the PBL between the pressure gradient force,Coriolis force and the eddy stress of the mean flow.

Thus we have, $0 = f(\bar{v} - \bar{v}_g) - \frac{\partial(\overline{u'w'})}{\partial z}$ ……(4.14)

$$0 = -f(\bar{u} - \bar{u}_g) - \frac{\partial(\overline{v'w'})}{\partial z} \quad \text{……(4.15)}$$

Parameterization of eddy stress in an unstably stratified PBL.

As we know in such case PBL is dominated by convective turbulence. Due to convection, the PBL is well mixed i.e. the mean quantities remain almost invariant with height in the PBL. In such case to a first approximation it is possible to treat the layer as a slab, in which mean horizontal wind, potential temperature remains invariant in the vertical and turbulent fluxes vary linearly with height. For simplicity, one can assume that at the top of CBL, turbulent vanishes. Observations indicate that the surface momentum flux can be represented by a bulk aerodynamic formula

$(\overline{u'w'})_s = -C_d\left(\sqrt{\bar{u}^2 + \bar{v}^2}\right)\bar{u}$ ….(4.16)

and $(\overline{v'w'})_s = -C_d\left(\sqrt{\bar{u}^2 + \bar{v}^2}\right)\bar{v}$ ….(4.17). So integrating equations (4.14) and (4.15) in the vertical between the bottom [generally taken at Z =0] and top of the boundary layer, we obtain

$\bar{v} = \frac{C_d}{f\,h}\left(\sqrt{\bar{u}^2 + \bar{v}^2}\right)\bar{u}$ ….(4.18)

And $\bar{u} = \bar{u}_g - \frac{C_d}{fh}$ …. (4.19), where h is the height of the boundary layer.

Parameterization of eddy stress in a stably stratified PBL.

In a stably stratified PBL, the mean quantities do vary in the vertical. In such case vertical eddy tranport of any quantity ‘S’ is parameterized using K-theory/similarity theory /flux-gradient theory.

According to this theory, vertical eddy transport of any physical quantity is proportional to the vertical gradient of the mean of that quantity and directed down the gradient, i.e.,

$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z}$ …. (4.19),

$$\overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z} \quad \text{.... (4.20)}$$

and $\overline{\theta' w'} = -K_h \frac{\partial \bar{\theta}}{\partial z}$ .... (4.21), where, K_m and K_h are constants, known as eddy coefficients. In this theory they are treated to be invariant in the vertical, which is a limitation of this theory.

Mixing length theory:

This theory was proposed by Prandtl. This theory is for a stably stratified PBL.

We consider an eddy in the PBL, initially embedded in the mean flow at the same level. If the eddy is displaced vertically, then it will carry the physical properties of the mean flow of the old level. After some eddy vertical displacement, say, $'l''$, the eddy mixes with the mean flow at a new level, imparting all its physical properties to it. This causes a fluctuation in the physical properties at the new level.

As per mixing length theory, fluctuation in the physical property is proportional to $'l'$ and to the vertical gradient of the mean of the physical property, i.e. for an arbitrary physical property 'S' we have $S' = -l' \frac{\partial \bar{S}}{\partial z}$. We know that in the PBL the order of magnitude of the vertical motion of mean flow is comparable with that of horizontal motion, so, $\bar{w} \approx |\bar{V}|$, where, $|\bar{V}| = \left(\bar{u}^2 + \bar{v}^2\right)^{\frac{1}{2}}$.

Hence, $\overline{S'w'} = -\overline{l'^2} \frac{\partial \bar{S}}{\partial z} \left| \frac{\partial \bar{V}}{\partial z} \right|$ .....(4.22).

Again from K- Theory, we have,

$$\overline{S'w'} = -K \frac{\partial \bar{S}}{\partial z}$$

Combing these two theories, we have, $K = \overline{l'^2} \left| \frac{\partial \bar{V}}{\partial z} \right|$ ....(4.23). The parameter $L = \sqrt{\overline{l'^2}}$, is known as mean mixing length. It is analogous to mean free path in the kinetic theory of gas. It is a measure of the eddy size.

Thus the value of K is large for large eddies and for large vertical shear of mean horizontal wind. Thus eddy transfer, is more for larger eddy greater vertical shear of horizontal wind.

Ekmann layer

It is also known as entrainment layer. In this layer there is approximately a balance between the pressure gradient force, coriolis force and eddy stress. Using the geostropic approximation at the top of PBL and from equations (4.19) & (4.20) we have

$$0 = f(\bar{v} - \bar{v}_g) + K_m \frac{\partial^2 \bar{u}}{\partial z^2} \text{ …..(4.24)}$$

and $0 = -f(\bar{u} - \bar{u}_g) + K_m \frac{\partial^2 \bar{v}}{\partial z^2}$ …..(4.25)

The above equations are solved using the following boundary conditions:

(1). $\bar{u}(z) = \bar{v}(z) = 0$ at $z = 0$

(2). As $z \to \infty, \bar{u} \to \bar{u}_g$ and $\bar{v} \to \bar{v}_g$.

Adding equation (4.24) with i ($= \sqrt{-1}$) times the equation (4.25) results into

$\frac{\partial^2 C}{\partial z^2} - \frac{i f}{K_m} C = \frac{-i f}{K_m} C_g$ ….(4.26), where, $C = u + iv$ and $C_g = u_g + iv_g$.

The general solution of (4.25) consists of two parts, Viz., the complementary function (CF) and the particular integral (PI).

$CF = A\exp[(1+i)\gamma z] + B\exp[-(1+i)\gamma z]$ …..(4.27), where, $\gamma = \sqrt{\frac{f}{2K_m}}$.

$PI = \frac{1}{D^2 - \frac{if}{K_m}}(-\frac{if}{K_m} C_g) = C_g$ …..(4.28).

Thus general solution is given by,

$C = A\exp[(1+i)\gamma z] + B\exp[-(1+i)\gamma z] + C_g$ ….(4.29).

The arbitrary constants A and B are determined from boundary conditions (1) and (2).

Accordingly, $A = 0$ and $B = -C_g$.

Hence, the particular solution is given by

$C = C_g[1 - \exp\{-(1+i)\gamma z\}]$ ….(4.30).

Now separation of the real and imaginary part on both sides of equation (4.30) results into,

$\bar{u} = \bar{u}_g[1 - e^{-\gamma z}\cos(\gamma z)] - \bar{v}_g e^{-\gamma z}\sin(\gamma z)$ …..(4.31)

$\bar{v} = \bar{u}_g e^{-\gamma z}\sin(\gamma z) + \bar{v}_g[1 - e^{-\gamma z}\cos(\gamma z)]$ …..(4.32)

The above two equations give the vertical profile of mean horizontal wind in the Ekmann layer.
From the above two equations it is evident that ,

$$R^2 = (\bar{u} - \bar{u}_g)^2 + (\bar{v} - \bar{v}_g)^2 = \left|\overline{V}_g\right|^2 e^{-2\gamma z}$$

$$\therefore R = \left|\overline{V}_g\right| e^{-\gamma z} \quad \text{.....(4.33).}$$

From (4.33) it is evident that if R [i.e., if $(\bar{u} - \bar{u}_g)$ and $(\bar{v} - \bar{v}_g)$] be plotted on a plane at different level, then after joining the points taken in order, we get a spiral, which is known as Ekmann Spiral.
If the axes of co-ordinates are rotated in such a way that , x-axis becomes parallel to the isobars, then $\frac{\partial \bar{p}}{\partial x} = 0$ and hence, $\bar{v}_g = \frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} = 0$. And then equations (4.31) and (4.32) further simplified to

$$\bar{u} = \bar{u}_g \left[1 - e^{-\gamma z}\cos(\gamma z)\right] \quad \text{.....(4.34)}$$

$$\bar{v} = \bar{u}_g e^{-\gamma z}\sin(\gamma z) \quad \text{......(4.35)}$$

Depth of Ekmann layer can be obtained from the following consideration:
At the bottom and top of Ekmann layer, $\bar{v} = 0$, which gives , from (4.35), $\sin(\gamma z) = 0 \Rightarrow z = 0$ & $z = \frac{\pi}{\gamma}$. These values of z correspond to bottom and top of Ekmann layer. Hence depth of this layer is $\frac{\pi}{\gamma}$.

Secondary circulation and Spin down.

We have seen that at the bottom and top of the Ekmann layer $\bar{v} = 0$ and at any intermediate level $\bar{v} \neq 0$. We know than $\bar{v}$ is the cross isobaric component of the mean flow.

Thus throughout the Ekmann layer there is a cross isobaric mass transport which causes convergence in a low pressure area and divergence in a high press area. This is known as frictional convergence.

Now in case of a low pressure area, the mass converged rises vertically and crosses the top of the Ekmann layer. Thus the mass from the Ekmann layer is being transported to the free atmosphere. This is known as Ekmann layer pumping.

The mass which rises vertically loses its vertical momentum after moving a distance in the vertical. The mass which losses its vertical momentum at some level, expands i.e. divergence. This divergence causes an anticyclonic circulation super imposed on the pre-existing cyclonic circulation associated with the low press area. The cyclonic circulation in this case is known as the primary circulation and the anticyclonic circulation is known as secondary circulation. Similar and opposite argument holds for a surface high also.

Now the super imposed secondary circulation, having sense opposite to that of primary circulation, reduces the speed of rotation of the primary circulation, is known as 'Spin down' process.

Mean motion in the layer adjacent to surface:

Skin layer is characterized by sheared flow forced by molecular viscosity. In this layer we introduce a quantity 'u_*' having the dimension of wind velocity. This is termed as friction velocity. Eddy stress in this layer is expressed in terms of this friction Velocity as follows:

$\overline{u'w'} = u_*^{\,2} \cos\mu$ ….(4.36)

$\overline{v'w'} = u_*^{\,2} \sin\mu$ …..(4.37), μ is the angle made by eddy stress vector with x-axis.

using flux gradient theory, we have,

$$K_m^{\,2}\left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right] = u_*^{\,4} \quad \text{….(4.38)}$$

Dimensional analysis of the above results into,

$K_m = k u_* z$ …..(4.39), k is called Von-Kerman constant.

Thus we have from equations (4.38) & (4.39),

$k u_* z \dfrac{\partial \bar{V}}{\partial z} = u_*^{\,2} \Rightarrow \dfrac{\partial \bar{V}}{\partial z} = \dfrac{u_*}{kz}$ ….(4.40), where, $\dfrac{\partial \bar{V}}{\partial z} = \sqrt{\left(\dfrac{\partial \bar{u}}{\partial z}\right)^2 + \left(\dfrac{\partial \bar{v}}{\partial z}\right)^2}$ is the magnitude of vertical shear of mean horizontal motion.

Integrating (4.40) vertically from $z = z_0$ to an arbitrary level, say, z, in PBL we obtain,

$\bar{V}(z) = \dfrac{u_*}{k}\ln\dfrac{z}{z_0}$ …..(4.41), where, $\bar{V}(z_0) = 0$ and z_0 is a constant, known as Roughness length. It may be interpreted physically as "From z = 0 to z = z_0, the surface is so rough that it does not allow any mean motion".

Oceanic Ekmann layer: For the oceanic Ekmann layer, the horizontal components of pressure gradient can be neglected as compared to vertical pressure gradient, because the horizontal pressure distribution over the ocean is almost uniform in absence of any system.

So, to a first degree approximation, there is a balance between Coriolis force and eddy stress, i.e., we have,

$$0 \approx f\bar{v} - \frac{\partial(\overline{u'w'})}{\partial z} \quad \text{….(4.42)}$$

$$0 \approx -f\bar{u} - \frac{\partial(\overline{v'w'})}{\partial z} \quad \text{…..(4.43).}$$

Again using K- theory for ocean, we have,

$\overline{u'w'} = -K_w \dfrac{\partial \overline{u}}{\partial z}$ and $\overline{v'w'} = -K_w \dfrac{\partial \overline{v}}{\partial z}$. Using these two results and by (4.43)+ i (4.42), we get,

$\dfrac{\partial^2 C}{\partial z^2} - \dfrac{if}{K_w} C = 0$, where, $C = \overline{u} + i\overline{v}$.

The general solution of the above 2nd order ordinary homogeneous differential equation may be given as,

$C = A\exp[(1+i)\gamma_w z] + B\exp[-(1+i)\gamma_w z]$ ….(4.44), where, $\gamma_w = \sqrt{\dfrac{f}{2K_w}}$ and A, B are arbitrary constants of integration, to be determined from the following boundary conditions:

BC 1: The ocean bottom is assumed to be at an infinite depth below mean sea level where the ocean current is assumed to be ceased, i.e., as $z \to -\infty$, both $\overline{u}(z)$ and $\overline{v}(z) \to 0$. This condition leads to $B = 0$ ….(4.45).

BC 2: At the ocean surface, stress exerted by surface wind on ocean is equal and opposite to that exerted by ocean on wind.

Now the components of surface wind stress exerted on ocean are respectively $-\rho_s \left(\overline{u'w'}\right)_{z=0} = \rho_s K_m \left(\dfrac{\partial \overline{u}}{\partial z}\right)_{z=0}$ and $-\rho_s \left(\overline{v'w'}\right)_{z=0} = \rho_s K_m \left(\dfrac{\partial \overline{v}}{\partial z}\right)_{z=0}$, where, ρ_s is air density at surface. Now from (4.34) and (4.35) we have, $\left(\dfrac{\partial \overline{u}}{\partial z}\right)_{z=0} = \left(\dfrac{\partial \overline{v}}{\partial z}\right)_{z=0} = \gamma \overline{u}_g$. Hence both the components of surface wind stress exerted on ocean are equal to $K_m \rho_s \gamma \overline{u}_g = \tau_0$ (say).

Now the components of ocean stress on surface wind are respectively, $-\rho_{ws} \left(\overline{u'w'}\right)_{z=0} = \rho_{ws} K_w \left(\dfrac{\partial \overline{u}}{\partial z}\right)_{z=0}$ and $-\rho_{ws} \left(\overline{v'w'}\right)_{z=0} = \rho_{ws} K_w \left(\dfrac{\partial \overline{v}}{\partial z}\right)_{z=0}$ where, ρ_{ws} is density of ocean water at surface, K_w is exchange coefficient for ocean water and $\overline{u}, \overline{v}$ are components of ocean current.

Hence following BC 2 we have, $\left(\dfrac{\partial C}{\partial z}\right)_{z=0} = -\dfrac{\tau_0}{K_w \rho_{ws}}(1+i)$ …(4.46). Using (4.44), (4.45) and (4.46), we have, $A = -\dfrac{\tau_0}{K_w \rho_{ws} \gamma_w}$ (Constant)……(4.47).

Hence, using (4.47) and (4.45) in (4.44). we have,

$$C(z)=\bar{u}(z)+i\bar{v}(z)=-\frac{\tau_0}{K_w\rho_{ws}\gamma_w}\exp[(1+i)\gamma_w z]=E\exp(\gamma_w z)[\cos(\gamma_w z+\pi)+i\sin(\gamma_w z+\pi)]$$

where, $E=\frac{\tau_0}{K_w\rho_{ws}\gamma_w}$.

Hence, the mean ocean current in the oceanic Ekmann layer is given by,

$$\bar{u}(z)=E\exp(\gamma_w z)Cos(\gamma_w z+\pi) \quad ....(4.48)$$

$$\bar{v}(z)=E\exp(\gamma_w z)Sin(\gamma_w z+\pi) \quad .....(4.49).$$

Conventionally, the Ekman layer depth h_E is defined as the depth where the current direction becomes exactly opposite to the surface current direction.

Hence we have,

$\hat{i}\bar{u}(h_E)+\hat{j}\bar{v}(h_E)=-\lambda[\hat{i}\bar{u}(0)+\hat{j}\bar{v}(0)]$; λ being a scalar constant.

Equating $\hat{j}$ component on both sides, we obtain, $E\exp(\gamma_w h_E)Sin(\gamma_w h_E+\pi)=0 \Rightarrow h_E=-\frac{\pi}{\gamma_w}$.

Now if M_x, M_y are respectively the wind driven mass transport in the oceanic Ekman layer along x & y axes respectively, then,

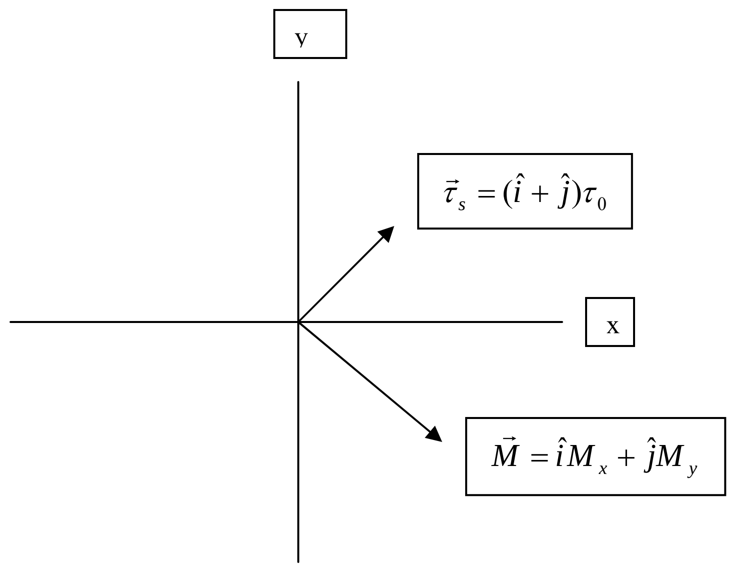
$M_x=\int_{-h_E}^{0}\rho_{ws}\bar{u}(z)dz$ and $M_y=\int_{-h_E}^{0}\rho_{ws}\bar{v}(z)dz$.

Hence, $M_x+iM_y=\rho_{ws}\int_{-h_E}^{0}(\bar{u}(z)+i\bar{v}(z))dz=\frac{e^{-\pi}\rho_{ws}(1-i)}{2\gamma_w}$.

Hence, $M_x=\frac{e^{-\pi}\rho_{ws}}{2\gamma_w}$ and $M_y=-\frac{e^{-\pi}\rho_{ws}}{2\gamma_w}=-M_x$.

Now surface wind stress vector $\vec{\tau}_s$ is given by, $\vec{\tau}_s=\tau_0(\hat{i}+\hat{j})$.

Orientation of the vector $\vec{M}$ and $\vec{\tau}_s$ are given in adjoining figure. From the figure and from the expression of these two vectors, it can clearly be shown that $\vec{\tau}_s$ is in the first quadrant making an angle 45^0 with positive direction of x-axis where as $\vec{M}$ lies in fourth quadrant making an 45^0 with positive direction of x-axis. Hence, $\vec{M}\perp\vec{\tau}_s$ and it is to the right of $\vec{\tau}_s$. Hence the wind driven mass transport in the oceanic Ekman layer is normal to the surface wind stress and it is to the right of surface wind stress in the Northern hemisphere.


y
$\vec{\tau}_s = (\hat{i} + \hat{j})\tau_0$
x
$\vec{M} = \hat{i}M_x + \hat{j}M_y$